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Some Models on McCoy Rings

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Abstract— The reliable way of exploring rings with involution, briefly *-rings, is to think about them within the category of *-rings with morphisms preserving involution. The main objective of this study is to address a new concept of generalized $* -$ McCoy $* -$rings and establish some of its basic properties and characterizations. Some examples and theorems are presented to explain this concept.

Key words—McCoy $* -$rings; reduced $* -$ rings; Armendariz $* -$ring; $* -$Armendariz $* -$ring

الملخص— الطريقة الموثوقة لاستكشاف الحلقات الالتفافية هي التفكير فيها ضمن فئة الحلقات ذات الأشكال التي تحافظ على الالتفاف. الهدف الرئيسي من هذه الدراسة هو تناول مفهوم جديد للحلقات الالتفافية تعرف بحلقات مكوي الالتفافية وتحديد بعض خصائصها الأساسية. وقد تم عرض بعض الأمثلة والنظريات لشرح هذا المفهوم.

الكلمات المفتاحية— حلقات مكوي؛ الحلقات الاختزالية؛ حلقات أرماندرير ؛ حلقات أرماندريز الإلتفافية.

1. Introduction

By a ring, researchers always mean an associative ring with identity. A ring R is said to be $* -$ring if on R there is a defined involution $*$. $* -$rings are objects of the category of rings with involution with morphisms also preserving involution. Therefore, the consistent way of investigating $* -$rings is to study them within this category as reported by a series of studies [1-7].

According to Nielsen **[8]**, a ring R is said to be a right McCoy ring when the equation $f(x)g(x) = 0$, where $f(x), g(x) \in R[x]/\{0\}$, implies that there exists a nonzero element $c \in R$ such that $f(x)c = 0$. The definition of a left McCoy ring is similar. If R is both a left and a right McCoy ring, then R is called a McCoy ring.

Recall that a ring R is called a reduced ring if it has no nonzero nilpotent elements. It is well known that if R is a reduced ring, then the following condition holds: $ab = 0$ implies $ba = 0$, for all $a, b \in R$, clearly reduced and commutative rings are McCoy rings [9,10]. A ring R is called Armendariz if whenever polynomials $f(x) = a_0 + a_1x + \ldots + a_nx^n, g(x) = b_0 + b_1x + \ldots + b_mx^m \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_ib_j = 0$ for each i, j. It is easy to see that all Armendariz rings are McCoy rings by definition.

Another generalization of a reduced $* -$ring is a $* -$Armendariz $* -$ring. A $* -$ring R is said to be a $* -$Armendariz $* -$ring if, whenever polynomials $f(x) = a_0 + a_1x + \cdots + a_mx^m, g(x) = b_0 +$

$b_1x + \cdots + b_nx^n \in R[x]$ satisfy $f(x)g(x) = f(x)g^*(x) = 0$, then $a_ib_j = 0$ (consequently $a_ib_j^* = 0$) for each i, j.

Throughout this paper, $M_n(R)$ will denote the full matrix ring of all $n \times n$ matrices over the ring R, while $T(R)$ ($T_{nE}(R)$) will denote the $n \times n$ upper triangular matrix ring (with equal diagonal elements) over R. Moreover, for a commutative ring R, the involution $\diamond$ defined on $T_{nE}(R)$ for $n > 2$ is given by replacing each entry with its involutive image and fixing the two diagonals, considering the right upper diagonal = left lower diagonal as symmetric ones and interchanging the symmetric elements about it. For $n = 2$ (trivial extension $T(R, R)$, the involution $\diamond$ is the adjoint involution **[11]**.

This study aims to introduce the notion of McCoy with involution, which generalize reduced McCoy and $*-$Armendariz. Moreover, this research explores the algebraic properties of these $*-$rings and their relationships with many other $*-$rings which have been studied previously.

2. $*-$McCoy $*-$Rings

This section introduces McCoy's condition for $*-$rings. If R is a $*-$ring, then the involution $*$ can naturally be extended to R[x] as:

$$\big(f(x)\big)^* = \left(\sum_{i=0}^n a_ix^i\right)^* = \sum_{i=0}^n a_i^*x^i \quad \text{for all} \quad f(x) \in R[x]$$

Definition: A $*-$ring R is called a $*-$McCoy if whenever the polynomials
$f(x) = a_0 + a_1x + \ldots + a_mx^m$ and $g(x) = b_0 + b_1x + \ldots + b_nx^n \in R[x] \setminus \{0\}$ satisfy $f(x)g(x) = f(x)g^*(x) = 0$, then $f(x)r = tg(x) = 0$ for some $0 \neq r, t \in R$ (consequently $g^*(x)t = 0$).

Since each McCoy $*-$ring is $*-$ McCoy and each $*-$Armendariz is $*-$ McCoy $*-$ring, but the converse is not true as shown by the following examples:

Example 1. According to [12], the ring $R = T_2(F)$, for some filed F, is neither left nor right McCoy. While R is $*-$McCoy ($*-$Armendariz) with involution $*: R \to R$ defined as $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}^* = \begin{pmatrix} c & -b \\ 0 & a \end{pmatrix}$ [13 Example 4].

Example 2. The $*-$ring $T_{4E}(R)$ is a McCoy [12, Lemma 2.1] and so $*-$MaCoy. Moreover, $T_{4E}(R)$ is not $*-$Armendariz [11, Example 5].

The question when a $*$-McCoy $*$-ring is McCoy has a partial answer in Proposition 1, where [11, Lemma 1] is needed, the question can be easily proved.

Lemma 1. [11, Lemma 1] Let R be a reduced $*-$ring and $f, g \in R[x]$ with $f(x) = \sum_{i=0}^m a_ix^i$ and $g\,(x) = \sum_{j=0}^n b_jx^j$. Then $f(gg^*) = f(gg^*)^* = 0$ if and only if $a_ib_jb_{k-(i+j)}^* = 0$ for all $0 \leq i, j \geq k, j \leq k \leq m + n$.

Proposition 1. Let R be a $*-$McCoy $*-$ring with proper involution, then R is McCoy.

Proof. Let $f(x)g(x) = 0$ for some $f(x), g(x) \in R[x]$. Then $0 = f(gg^*) = f(gg^*)^*$ implies $a_ic_k = 0$, since R $*-$McCoy and $c_k = \sum_{i=0}^k rb_{k-j}^*$. Hence $\sum_{i=0}^k \sum_{j=0}^k a_i\big(rb_{k-(i+j)}^*\big)$ and consequently

$a_i r b^*_{k-(i+j)} = 0$. Now $(a_i r)(a_i r)^* = a_i r r^* a_i^* = 0$. Since $*$ is proper then $a_i r = 0$, which implies $\sum_{i=0}^{m} a_i r x^i = f(x)r = 0$, this means that R is McCoy.

Given that a reduced *-ring is McCoy [14], the following can be stated:

Proposition 2. Each reduced $*-$ring is $*-$ McCoy. The converse of the previous proposition is not true as shown by the following example:

Example 3. The $*$-ring $R = \begin{pmatrix} 0 & F \\ 0 & 0 \end{pmatrix}$, with adjoint involution $*$ defined by:
$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}^* = \begin{pmatrix} c & -b \\ 0 & a \end{pmatrix}$ is $*-$Armendariz, thus R a $*-$McCoy. Moreover, R is not reduced since the nonzero matrix $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ satisfies $A^2 = 0$.

3. Matrix $*-$rings over $*-$McCoy $*-$rings

This section will shed light on the extension of McCoy property for a $*-$subring of the $*-$ring of upper triangular matrices. For the $\diamond-$ring $T_{3E}(R)$, of upper triangular $n \times n$ matrices over a $*-$ring R, it can be extended the involution $\diamond$ of R for $T_{3E}(R)$, This involution will be considered here and in the next proposition.

Proposition 3. A $*-$ring R is $*-$McCoy if and only if the $\diamond-$ring $T_{3E}(R)$ is $\diamond-$McCoy.

Proof

$\Rightarrow$ Let $f(x), g(x) \in T_{3E}(R)[x]$ such that

$$f(x) = \sum_{i=0}^{m} \begin{pmatrix} a_i & b_i & c_i \\ 0 & a_i & d_i \\ 0 & 0 & a_i \end{pmatrix} x^i, g(x) = \sum_{j=0}^{n} \begin{pmatrix} a'_j & b'_j & c'_j \\ 0 & a'_j & d'_j \\ 0 & 0 & a'_j \end{pmatrix}, \text{where}$$

$f_0(x) = \sum_{i=0}^{m} a_i x^i, f_1(x) = \sum_{i=0}^{m} b_i x^i, f_2(x) = \sum_{i=0}^{m} c_i x^i, f_3(x) = \sum_{i=0}^{m} d_i x^i, g_0(x) = \sum_{j=0}^{n} a'_j x^j, g_1(x) = \sum_{j=0}^{n} b'_j x^j, g_2(x) = \sum_{j=0}^{n} c'_j x^j, g_3(x) = \sum_{j=0}^{n} d'_j x^j$.
Assume that $f(x)g(x) = f(x)g^{\diamond}(x) = 0$, and $f(x), g(x) \neq 0$.

CASE 1. If $f_0(x) \neq 0, g_0(x) \neq 0$, then $h_{11}(x) = f_0(x)g_0(x) = f_0(x)g_0^*(x) = 0$. Since R is $*-$McCoy, there exist $r, t \in R \setminus \{0\}$ such that $f_0(x)r = 0$ and $tg_0(x) = 0$. Let $A = E_{13}(r), B = E_{13}(t)$. Then $f(x)A = 0$ and $Bg(x) = 0$.

CASE 2. If $f_0(x) \neq 0, g_0(x) = 0$, then there exists $g_2(x) \neq 0$, such that $g_1(x) = g_3(x) = 0$; since $g_2(x) \neq 0$. So $f_0(x)g_2(x) = f_0(x)g_2^*(x) = 0$. There exists $r, t \in R \setminus \{0\}$ such that $f_0(x)r = 0, tg_2(x) = 0$ because R is $*-$McCoy. Let $A = E_{13}(r)$. Then $Ag(x) = g(x)A = 0$.

CASE 3. If $f_0(x) = 0$, $g_0(x) \neq 0$, then there exist $A, B \in T_{3E}(R) \setminus \{0\}$ such that $Ag(x) = g(x)B = 0$. The proof is similar to Case 2.

CASE 4. If $f_0(x) = 0$, $g_0(x) = 0$, then for any $r \in R \setminus \{0\}$, let $A = E_{13}(r)$. It is obvious that $Ag(x) = g(x)A = 0$. Therefore, $T_{3E}(R)$ is $\diamond-$McCoy for any case.
$\Leftarrow$ Assume that $f_0(x)g_0(x) = f_0(x)g_0^*(x) = 0$, where

$f_0(x) = \sum_{i=0}^{m} a_i x^i \neq 0\, , g_0(x) \sum_{j=0}^{n} a'_j x^j \neq 0\, , a_i, a'_j \in R$.

Let $f(x) = \sum_{i=0}^{m} \begin{pmatrix} a_i & a_i & a_i \\ 0 & a_i & a_i \\ 0 & 0 & a_i \end{pmatrix} x^i, g(x) = \sum_{j=0}^{n} \begin{pmatrix} a'_j & a'_j & a'_j \\ 0 & a'_j & a'_j \\ 0 & 0 & a'_j \end{pmatrix}$ for any $i = 0,1,\cdots, m, j = 0,1,\cdots n$. Then

$f(x)g(x) = \begin{pmatrix} f_0(x) & f_0(x) & f_0(x) \\ 0 & f_0(x) & f_0(x) \\ 0 & 0 & f_0(x) \end{pmatrix} \begin{pmatrix} g_0(x) & g_0(x) & g_0(x) \\ 0 & g_0(x) & g_0(x) \\ 0 & 0 & g_0(x) \end{pmatrix}$. Hence, there exists $A = \begin{pmatrix} s & s_{12} & s_{13} \\ 0 & s & s_{23} \\ 0 & 0 & s \end{pmatrix} \in T_{3E}(R) \setminus \{0\}$ such that $f(x)A = 0$ because $T_{3E}(R)$ is $\diamond$ −McCoy. If $s \neq 0$, then $f_0(x)s = 0$. If $s = 0$, then there exists $r_{ij} \neq 0$, such that $s_{23} = 0$. We also have $f(x)s_{12}$, $f(x)s_{13} = 0$. Similarly, there exists $t \in R \setminus \{0\}$ such that $tg_0(x) = 0$. Thus, R is $*$ −McCoy.

Corollary 1. A $*$ −ring R is $*$ −McCoy if and only if the $\diamond$ −ring T(R,R) with adjoint involution $\diamond$ is $\diamond$ −McCoy.

Proposition 3 suggests that $T_{nE}(R)$ could also be $\diamond$ −McCoy for all $n \geq 4$. However, the following example rules out this possibility.

Example 4. Consider $T_{4E}(R)$ over a commutative $*$ −McCoy $*$ −ring R and let

$$f(x) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} x, g(x) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} x,$$

satisfy $f(x)g(x) = f(x)g^*(x) = 0$, while $f(x) \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \neq 0$.

The full matrix $M_n(R)$ over a $*$ −ring R with transpose involution is not $*$ −McCoy, for $n \geq 3$, according to the following examples:

Example 5. The $*$ −ring $M_3(R)$ is not $*$ −McCoy. Indeed, the polynomials

$$f(x) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} x, g(x) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} x,$$ satisfy

$f(x)g(x) = f(x)g^*(x) = 0$, while $f(x) \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \neq 0$.

Example 6. The $*-$ring $M_4(R)$ is not $*-$McCoy. Indeed, the polynomials f(x),g(x) in Example 4, satisfy $f(x)g(x) = f(x)g^{*}(x) = 0$, while $f(x)\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \neq 0$

4. Extensions of $*-$McCoy property

In this section, the $*-$McCoy property is extended to several known extensions, such as the polynomial $*-$ring $R[x]$, the Laurent polynomial $*-$ring $R[x, x^{-1}]$, the localization $S^{-1}R$ of R at S and from Ore $*-$ring to its classical Quotient Q.

Let R be a $*-$ring and T be a multiplicatively closed subset of R consisting of nonzero central regular elements, then the localization of R to S is the $*-$ring $S^{-1}R = \{u^{-1}a \mid u \in S, a \in R\}$, with involution $*$ defined in [11] as:

$$(u^{-1}a)^{*} = u*^{-1}a*.$$

Proposition 4. A *-ring R is $*-$McCoy if and only if $S^{-1}R$ is $*-$McCoy.

Proof

$\Rightarrow$ Let R be a $*-$McCoy $*-$ring and $F(x)G(x) = F(x)G^{*}(x) = 0$ with

$F(x) = \sum_{i=0}^{m} \alpha_i x^i, G(x) = \sum_{j=0}^{n} \beta_j x^j \in S^{-1}R[x]$, where $\alpha_i = u^{-1}a_i$; $\beta_j = v^{-1}b_j$ and

$a_i;\ b_j \in R[x] \setminus \{0\}, u;\ v \in S$. Hence

$$F(x)G(x) = (u^{-1}a_0 + u^{-`1}a_1x + \cdots + u^{-1}a_mx^m)(v^{-1}b_0 + v^{-1}b_1x + \cdots + v^{-1}b_nx^n)$$
$$= u^{-1}v^{-1}a_0b_0 + u^{-1}v^{-1}(a_0b_1 + a_1b_0)x + \cdots$$
$$+ u^{-1}v^{-1}(a_0b_n + \cdots + a_mb_0)x^{m+n}$$
$$= (vu)^{-1}\big(a_0b_0 + (a_0b_1 + a_1b_0)x + \cdots + (a_0b_n + \cdots + a_mb_0)\big)x^{m+n}$$
$$= (vu)^{-1}f(x)g(x) = 0;$$

$$F(x)G^{*}(x) = (u^{-1}a_0 + u^{-`1}a_1x + \cdots + u^{-1}a_mx^m)(v^{-1^{*}}b_0^{*} + v^{-1^{*}}b_1^{*}x + \cdots + v^{-1^{*}}b_n^{*}x^n)$$
$$= u^{-1}v^{-1^{*}}a_0b_0^{*} + u^{-1}v^{-1^{*}}(a_0b_1^{*} + a_1b_0^{*})x + \cdots$$
$$+ u^{-1}v^{-1^{*}}(a_0b_n^{*} + \cdots + a_mb_0^{*})x^{m+n}$$
$$= (v^{*}u)^{-1}(a_0b_0^{*} + (a_0b_1^{*} + a_1b_0^{*})x + \cdots + (a_0b_n^{*} + \cdots + a_mb_0^{*})x^{m+n})$$
$$= (v^{*}u)^{-1}f(x)g^{*}(x) = 0;$$

since S is contained in the center of R, so $f(x)g(x) = f(x)g^{*}(x) = 0$. By hypothesis then there exists $r, t \in R$, such that $f(x)r = tg(x) = 0$ which implies $F(x)r = tG(x) = 0$. Therefore $S^{-1}R$ is $*-$McCoy.

$\Leftarrow$ Let $f(x) = \sum_{i=0}^{m} a_ix^i, g(x) = \sum_{j=0}^{n} b_jx^j \in R[x] \setminus \{0\}$ such that $f(x)g(x) = f(x)g^{*}(x) = 0$. Then there exists a nonzero element $\alpha \in S^{-1}R$ such that $f(x)\alpha = 0$ since $S^{-1}R$ is $*-$McCoy. We can assume $\alpha = u^{-1}a$ for some $a \in R[x] \setminus \{0\}$ and $u \in S$. Then $f(x)au^{-1} = f(x)\alpha = 0$ implies that $f(x)a = 0$. Therefore, R is $*-$McCoy.

From Proposition 4, the following result is straightforward.

Corollary 2. For a $*-$ring R, $R[x]$ is $*-$McCoy if and only if $R[x,x^{-1}]$ is $*-$McCoy.

Proof.

Clearly, $S = \{1, x, x^2, \cdots\}$ is a multiplicatively closed subset of $R[x]$. Since $R[x,x^{-1}] = R[x]_T$, it follows that $R[x,x^{-1}]$ is $*-$McCoy by Proposition 4.

Recall that a ring R is called right Ore if given $a, b \in R$ with b regular there exist $a_1, b_1 \in R$ with b_1 regular such that $ab_1 = ba_1$. Left Ore is defined similarly and R is Ore ring if it is both right and left Ore. For $*-$rings, right Ore implies left Ore and vice versa. It is a known fact that R is Ore if and only if its classical quotient ring Q of R exists and for $*-$rings, $*$ can be extended to Q by $(a^{-1}b)^* = b^*(a*)^{-1}$ (see [15, Lemma 4]).

Proposition 5. Let R be an Ore $*-$ring and Q be its classical quotient $*-$ring, then R is $*-$McCoy if and only if Q is $*-$McCoy.

Proof

⇒ Let R be a $*-$McCoy $*-$ring and $F(x)G(x) = F(x)G^*(x) = 0$ with $F(x) = \sum_{i=0}^{m} \alpha_i x^i, G(x) = \sum_{j=0}^{n} \beta_j x^j \in Q[x],$. From [14, Theorem 12], we may assume that $\alpha_i = a_i u^{-1}$; $\beta_j = b_j v^{-1}$ where a_i; $b_j \in R$ for all $i, j, u, v \in R$ are regular and for each j there exist $c_j, c_j^*, d_j^* \in R$, regular $w, z^* \in R$ such that $u^{-1}b_j = c_j w^{-1}, u^{-1}d_j^* = c_j^* w^{-1}, v^{*^{-1}} b_j^* = d_j^* z^{*^{-1}}$. Put $f(x) = \sum_{i=0}^{m} a_i x^i, g(x) = \sum_{j=0}^{n} c_j x^j \in R[x]$, then

$$0 = F(x)G(x) = \sum_{i=0}^{m}\sum_{j=0}^{n} \alpha_i \beta_j x^{i+j} = \sum_{i=0}^{m}\sum_{j=0}^{n} a_i \left(u^{-1}b_j\right) v^{-1} x^{i+j}$$

$$= \sum_{i=}^{m}\sum_{j=}^{n} a_i c_j (vw)^{-1} x^{i+j} = f(x)g(x)(vw)^{-1} \text{and}$$

$$0 = F(x)G^*(x) = \sum_{i=0}^{m}\sum_{j=0}^{n} \alpha_i \beta_j^* x^{i+j} = \sum_{i=0}^{m}\sum_{j=0}^{n} a_i u^{-1} \left(b_j v^{-1}\right)^* x^{i+j}$$

$$= \sum_{i=0}^{m}\sum_{j=0}^{n} a_i u^{-1} (v^*)^{-1} b_j^* x^{i+j}$$

$$= \sum_{i=0}^{m}\sum_{j=0}^{n} a_i \left(u^{-1} d_j^*\right) z^{*^{-1}} x^{i+j}$$

$$= \sum_{i=0}^{m}\sum_{j=0}^{n} a_i c_j^* (z^* w)^{-1} x^{i+j}$$

$$= f(x)g^*(x)(z^* w)^{-1}.$$

Hence $f(x)g(x) = f(x)g^*(x) = 0$ in $R[x]$. Since R is $*-$McCoy, $f(x)r = tg(x) = 0$, we have $F(x)r = \sum_{i=0}^{m} \alpha_i x^i r = \sum_{i=0}^{m} a_i u^{-1} x^i r = \sum_{i=0}^{m} a_i x^i r u^{-1} = f(x) r u^{-1} = 0 = tG(x)$, for all i and Q is $*-$McCoy.

⇐ Let $f(x) = \sum_{i=0}^{m} a_i x^i, g(x) = \sum_{j=0}^{n} b_j x^j \in R[x] \setminus \{0\}$ such that $f(x)g(x) = f(x)g^*(x) = 0$. Then there exists a nonzero element $\alpha \in Q$ such that $f(x)\alpha = 0$ since Q is $*-$McCoy. Because Q is a classical right quotient $*-$ring, we can assume $\alpha = au^{-1}$ for some $a \in R[x] \setminus \{0\}$ and regular element u. Then $f(x)au^{-1} = f(x)\alpha = 0$ implies that $f(x)a = 0$. Therefore, R is $*-$McCoy.

Conclusion

In conclusion, this study established the concept of $*-$McCoy $*-$rings by applying the theory of $*-$ring to McCoy ring. The basic operations on this concept were defined along with several examples. Subsequently, theorems related to these operations were proven. Finally, algebraic relations between this ring and many other rings were presented.

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